**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Sol :- Since work being 10 min after the car is dropped, the time left to complete work 50 mins, Probability that service manager cannot meet his commitment =P(X>50)=1-Pr(x<=50)

Convert 50 to z-score

Standard normal variable Z=(X- )/=(x-45)/8

P(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625)=0.73237=73.237%

Probability that service manager will not meet his commitment is

100-73.237=26.763%=0.2676

The answer is B.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Sol :- 0.3413447460685429

True

So, the statement “More employees at the processing center are older than 44 than between 38 and 44”.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol :- 36.484487890347154

True

The statement of “training program for employees under the age of 30 at the center would be expected to attract about 36 employees”.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Sol :- Here X1 and X2 are two independent random variables than

X1+X2 ~N(μ+ μ, σ2+σ2)andX1-X2~N(μ-μ, σ2σ2)

2X1~N(2μ,2σ2)

2X1-(X1+X2) = N(2μ,2σ2)-N(μ+ μ, σ2+σ2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Sol :- The probability of getting value between a & b is 0.99

So, the probability of getting value outside a & b is 1-0.99=0.01

The probability towards left of a = -0.01/2 = -0.05

The probability towards right of b = 0.01/2 = 0.05

By finding standard normal variable (z), need to calculate x.

Z\* σ+ μ=x

-(-2.57)\*20+100=151.4

(-2.57)\*20+100=48.6

D is the correct answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Sol :- Profit1+Profit2~N(5+7, 32+42)=Profit~N(12,5)

1. Range = 12-19.6x5,12+ 1.96 x5

=$2.2, $22.8

=Rs.99, Rs.1026

1. P(Z<=(p-12)/5)=0.05

P-12/5=-1.644

P=12-8.22 = $3.78 = rs.170.1

1. When profit is less than 0 then loss

p-12/5=-1.644

p= 12-8.22 - $3.78 = Rs.170.1

1. The first division of company, thus have large probability of making loss in a given year.